Stagnation-point flow of a micropolar fluid over a nonlinearly stretching surface with suction

M. Jayachandra Babu[1] C S K Raju[2] Dr.N.Sandeep[3*]

Abstract - In this study we analyzed the stagnation-point flow of a micropolar fluid towards a nonlinearly stretching surface with suction/injection effects. The governing equations of the flow and heat transfer are transformed in to system of nonlinear ordinary differential equations by using similarity transformation and then solved numerically using bvp4c technique with Matlab Package. The influence of non-dimensional governing parameters like viscous dissipation parameter, suction/injection parameter, buoyancy parameter ,and material parameter on velocity and temperature profiles along with friction factor and heat transfer rate was discussed and presented through graphs and tables.

Index terms: Stagnation-point flow, radiation, dissipation, micropolar fluid, suction, stretching, micro rotation.

1 INTRODUCTION

Many researchers have attracted by micropolar fluids because of the equation governing the flow of micropolar fluid involves a micro rotation vector and a gyration parameter in addition to the definitive velocity vector field. These parameters plays major role in engineering and allied areas. The modeling of micropolar fluids was initially introduced by Ernigen [1]. MHD flow of a micropolar fluid near a stagnation point towards a non-linear stretching sheet was discussed by Hayat et al. [2]. The similar type of study over a shrinking surface was discussed by Lok et al. [3]. Papautsky et al. [4] studied micropolar fluid flow over a rectangular micro Kumar [5] explains the heat and mass transfer in micropolar fluid by considering stretching sheet using finite element analysis. Influence of suction and blowing in micropolar fluid flow over a stretching surface was discussed by Hassanienet al. [6]. Through a moving porous medium heat transfer in flow was discussed by Takhan et al. [7]. Radiation and chemical reaction effects on flow through vertical plate were discussed by John et al. [8]. Sandeep et al. [9] discussed radiation effect on unsteady natural convective flow of a nanofluid over a vertical plate. Ramana Reddy et al.[10] discussed influence of radiation along with chemical reaction on dusty fluid flow over a vertical channel. Attia [11] observed the heat generation and the effect of uniform suction/blowing in non-Newtonian micropolar fluid flow. Ahmadi[12] presented a self-similar solution for boundary layer flow in micropolar fluids past a semi-infinite plate.

- *M. Jayachandra Babu, pursuing ph.d in Fluid Dynamics Division, VIT University, Vellore -632014, India. Email: jayamacharla@gmail.com*
- *CSK Raju, pursuing ph.d in Fluid Dynamics Division, VIT University, Vellore -632014, India. Email: sivaphd90@gmail.com*
- *Dr. N. Sandeep, Assistant Professor, Fluid Dynamics Division, VIT University, Vellore-632014, India. Email:*nsreddy.dr@gmail.com

Radiation and magneticfield effects of a nanofluid past a vertical plate was discussed by Mohan Krishna et al. [13]. Aman et.al [14] investigated the MHD stagnation point flow over a vertical plate by considering the convective surface boundary conditions. Nazar et.al [15] analyzed the behavior of steady-state stagnation point flow of a micropolar fluid over a stretching sheet. Das [16] discussed the MHD mixed convection stagnation point flow of micropolar fluids past a shrinking sheet.Ziabakhsh et al. [17] derived an analytical solution to the non-Newtonian micropolar fluid by considering suction/blowing effects and heat generation.

Zaimi and Ishak [18] studied the heat flow and heat transfer characteristics of stagnation point flow of a micropolar fluid past a shrinking/stretching sheet.Zheng et al. [19] derived dual solutions for micropolar fluid flow and its radiative heat transfer past stretching/shrinking sheet. Radiation and chemical reaction effects on MHD convective flow over a vertical porous plate were discussed by Sandeep et al. [20]. Vyas [21] studied MHD boundary layer flow over a nonlinear stretching sheet in presence of radiation and viscous dissipation. Ishak et al. [22] analyzed stagnation point flow of a micropolar fluid towards a vertical permeable surface. Inclined magneticfiled and radiation effects on a dusty viscous fluid flow between parallel flat plates were discussed by Sandeep and Sugunamma [23]. The influence of thermal radiation and viscous dissipation on a flow past a vertical plate in porous medium was discussed by Sugunamma et al. [24].MHD nanofluid flow past a moving vertical plate in porous medium with radiation and soret effects was analyzed by Raju et al.[25]

In present study we discussed the stagnation-point flow of a micropolar fluid towards a nonlinearly stretching surface with suction and injection effects. The governing equations of the flow and heat transfer are transformed in to system of nonlinear ordinary differential equations by using similarity transformation and the solved numerically. The influence of non-dimensional governing parameters on velocity and temperature profiles along with friction factor and heat transfer rate was discussed and presented through graphs and tables.

2 MATHEMATICAL MODELLING

Consider a steady two-dimensional flow of an incompressible stagnation point flow of a micro polar fluid over nonlinearly shrinking sheet in presence of radiation, dissipation with suction/injection effects. We considered a stretching velocity $u_w(\mathrm{x})$, free stream velocity $u_e(\mathrm{x})$,temperature of the surface is $T_w(\mathrm{x})$ and T_∞ is the ambient temperature. Here $T_w(\mathrm{x}) = \mathrm{T}_\infty + cx^{2n}$, $u_e(\mathrm{x}) = ax^n$ and $u_w(\mathrm{x}) = bx^n$, where n, a, b *and* c are positive constants. Under the above assumptions, the governing equations for the flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = u_e\frac{du_e}{\partial x} + \frac{\mu+\kappa}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} + g_e\beta(T - T_\infty) \quad (2)$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\frac{\partial^2 N}{\partial y^2} - \kappa\left(2\mathrm{N} + \frac{\partial u}{\partial y}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu+\kappa}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (4)$$ with

the boundary conditions

$$u = u_w,\ v = v_w,\ N = -m\frac{\partial u}{\partial y}, T = T_\infty \text{ at } y = 0$$

$$u \to u_e,\ \mathrm{N} \to 0,\ \mathrm{T} \to T_\infty \text{ as } \mathrm{y} \to \infty \qquad (5)$$

where u and v are the velocity components in the x and y directions respectively, T is the fluid temperature, μ is the dynamic viscosity, N is the component of the micro rotation vector normal to the xy plane, κ is the vortex viscosity, j is the micro inertia density, ρ is the density, k is the thermal conductivity, σ^* is the Boltzmann constant, k^* is the Roseland mean absorption coefficient, γ is the spin gradient viscosity and c_p is the specific heat at constant pressure, m is a constant with $0 \le m \le 1$, v_w is suction/injection parameter. We assumed that $\gamma = (\mu + \kappa/2)j = \mu(1 + K/2)j$, where $K = k/\mu$ is the micropolar or material parameter.

We now introduce the following similarity variables to get the similarity transformation for the equations (1) to (4) subject to the boundary conditions (5)

$$\eta = \sqrt{\frac{u_e}{\upsilon x}}y, \psi = \sqrt{\upsilon x u_e}\, f(\eta),\ N = u_e\sqrt{\frac{u_e}{\upsilon x}}h(\eta),$$

$$h(\eta) = -\frac{1}{2}f''(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (6)$$

Where prime denotes a differentiation with respect to η (the similarity variable) and ψ is the stream function, which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, that satisfied the equation (1). The quantity $\upsilon = \mu/\rho$ is the kinematic viscosity, f is the dimensionless stream function, θ is the dimensionless temperature and h is the dimensionless micro rotation. Substituting (6) into (2)-(4), (2) and (3) are transformed into a single nonlinear ordinary differential equation (7) while the energy equation (4) reduces to (8) as follows:

$$\left(1 + \frac{K}{2}\right)f''' + \frac{n+1}{2}ff'' - nf'^2 + n + Gr_x\theta = 0 \quad (7)$$

$$\frac{1}{\Pr}(1+R)\theta'' + \frac{n+1}{2}f\theta' - 2nf'\theta + (1+K)Ecf''^2 = 0 \quad (8)$$

With the transformed boundary conditions

$$f(0) = f_w,\ f'(0) = \varepsilon,\ \theta(0) = 1$$

$$f'(\eta) \to 1,\ \theta'(\eta) \to 0 \text{ as } \eta \to \infty \qquad (9)$$

Where Pr is the Prandtl number, Ec is the Eckert number, R is the radiation parameter and $\varepsilon > 0$ is the stretching parameter, f_w is the suction/injection parameter, $f_w > 0$ for suction and $f_w < 0$ for injection, these are given as

$$\Pr = \frac{\mu c_p}{k}, Ec = \frac{a^2}{cc_p},\ Gr_x = \frac{g_e\beta x}{a^2},\ \ \varepsilon = \frac{b}{a} \quad (10)$$

For engineering interest the local skin friction coefficient and Nusselt number are given by

$$C_f \mathrm{Re}_x^{1/2} = \left(\frac{1+K}{2}\right)f''(0), Nu_x \mathrm{Re}_x^{-1/2} = -\theta'(0) \ (11)$$

3. RESULTS AND DISCUSSION

Equations (7) and (8) with the boundary conditions (9) have been solved numerically using bvp4c with Matlab package. For numerical results we considered the non-dimensional parameter valuesas$Pr = 1, R = K = 0.5, Ec = 0.1, \varepsilon = 2 = Gr\ and\ n = 1$.these values are kept as constant in entire study except the varied parameters as shown in figures and tables. The results obtained shows the influences of the non-dimensional governing parameters, namely radiation parameter, viscous dissipation parameter and material parameter and n on velocity and temperature profiles along with friction factor and Nusselt number.

Fig. 1 depicts the influence of material parameter on velocity profiles. It is evident from figure that an enhancement in material parameter improves the velocity profiles of the flow. It is due to the fact that increase in material parameter increases the velocity boundary layer thickness. Figs. 2 and 3 illustrate the influence of velocity and temperature profiles for various values of n. It is observed from figures that increase in the value of n leads to decrease in the velocity as well as temperature profiles of the flow.

Figs. 4 and 5 show the effect of buoyancy parameter on velocity and temperature profiles. It is clear from the figure that a rise in the value of buoyancy parameter enhances the velocity profiles and depreciates the temperature profiles. But increase in Buoyancy parameter enhances the heat transfer

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rate.Figs 6 and 7 display the influence of suction/injection parameter on velocity and temperature profiles of the flow. It is observed from the figure that an improvement in the suction/injection parameter depreciates the velocity and temperature profiles of the. Figs. 8 and 9 illustrate the effect of Eckert number on velocity and temperature profiles. It is clear that increase in dissipation parameter enhances the velocity and temperature profiles of the flow. This agrees the general physical behavior that enhancement in viscous dissipation improves the thermal conductivity of the flow.

Table 1 shows the influence of various non-dimensional governing parameters on skin friction coefficient and Nusselt number. It is evident from the table that increase in the material parameter enhances the friction factor and reduces the Nusselt number. Increase in the value of n depreciates the skin friction coefficient and improves the heat transfer rate. Enhancement in buoyancy parameter, suction parameter improves the heat transfer rate.

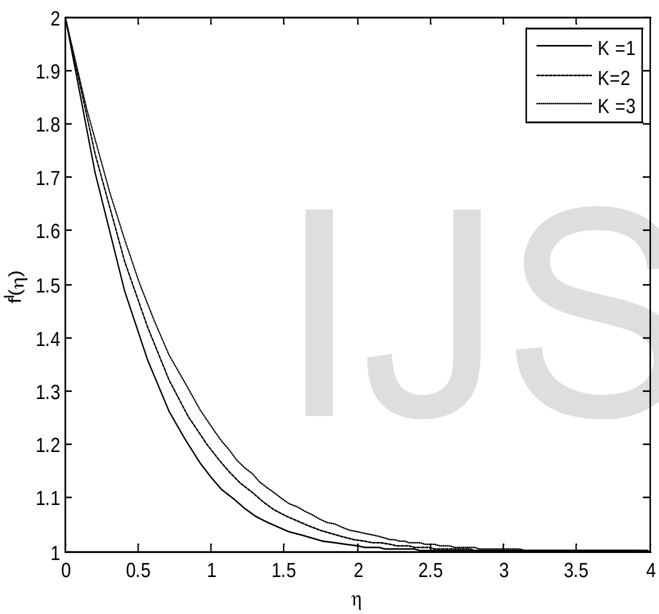

Fig.1 Velocity profiles for different values of K

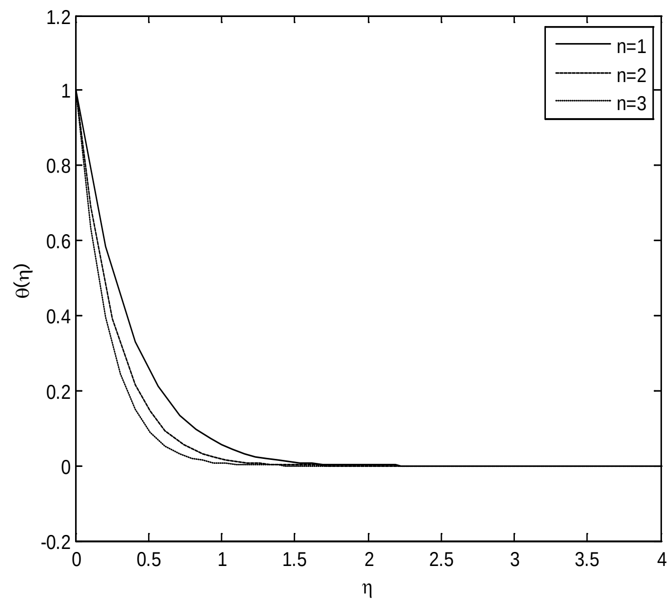

Fig.2 Temperature profiles for different values of n

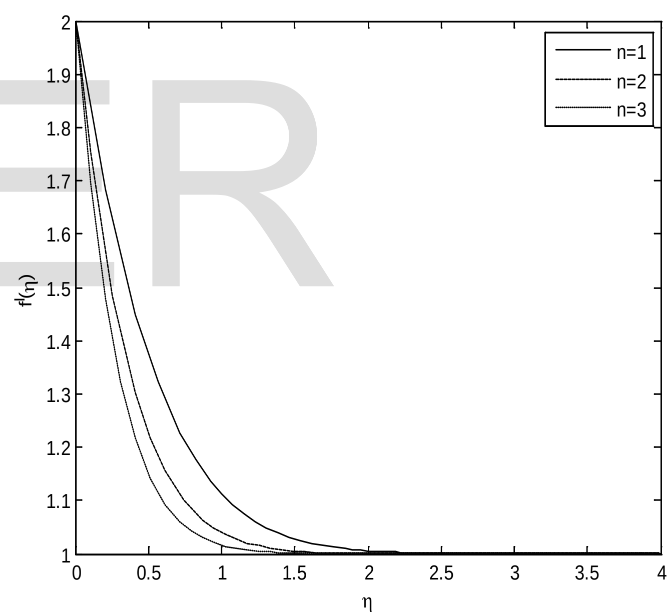

Fig.3 Velocity profiles for different values of n

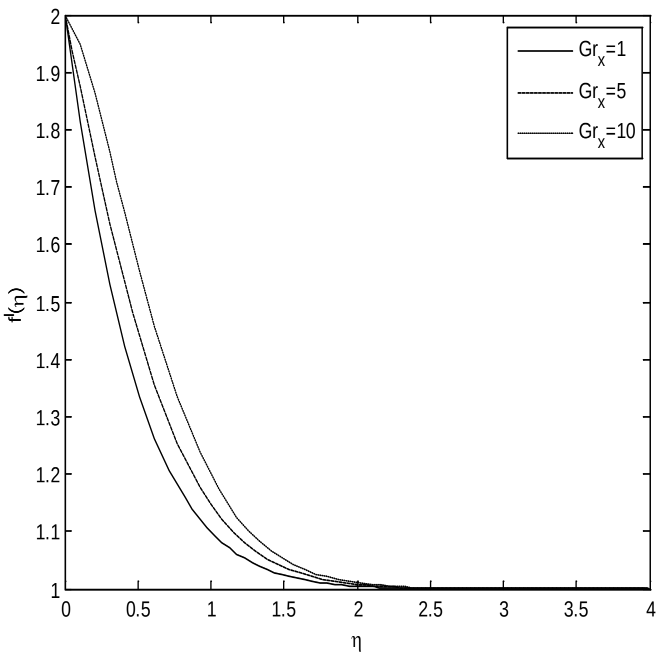

Fig.4Velocity profiles for different values of Gr_x

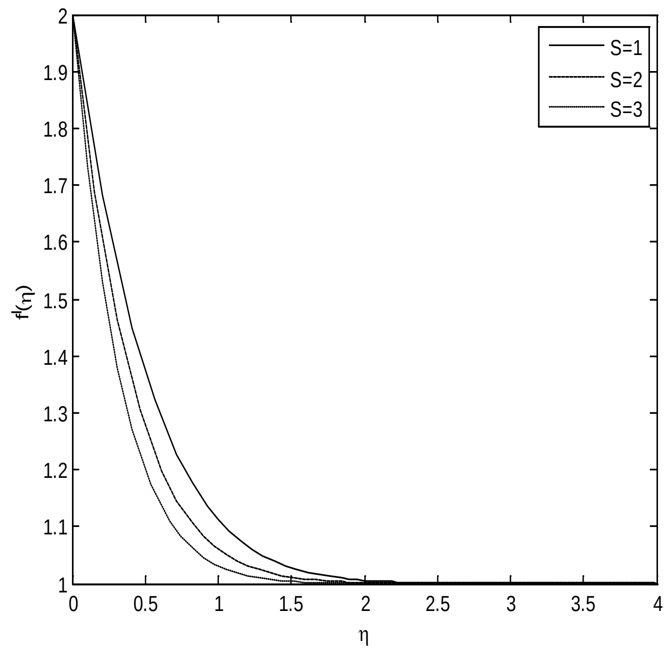

Fig.6Velocity profiles for different values of S

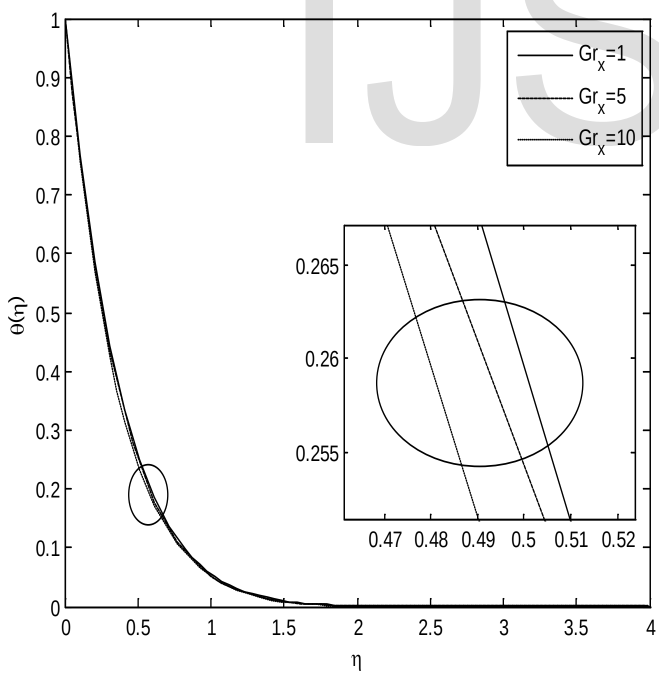

Fig.5Temperature profiles for different values of Gr_x

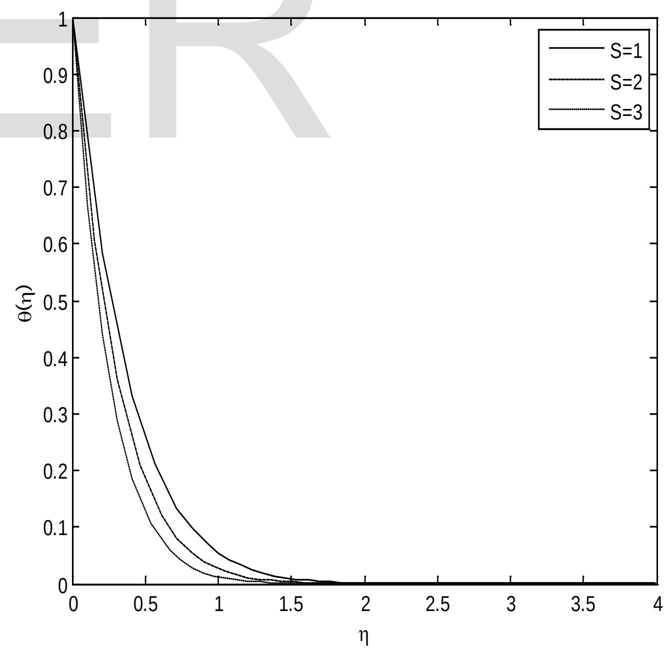

Fig.7Temperature profiles for different values of S

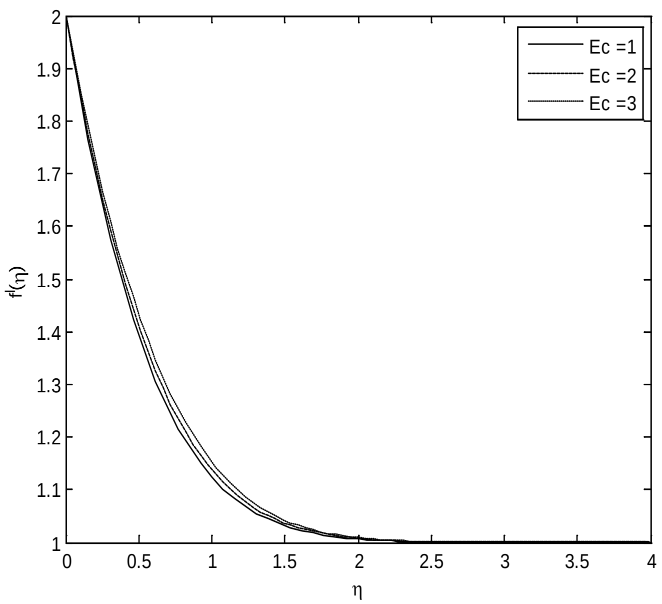

Fig.8 Velocity profiles for different values of Ec

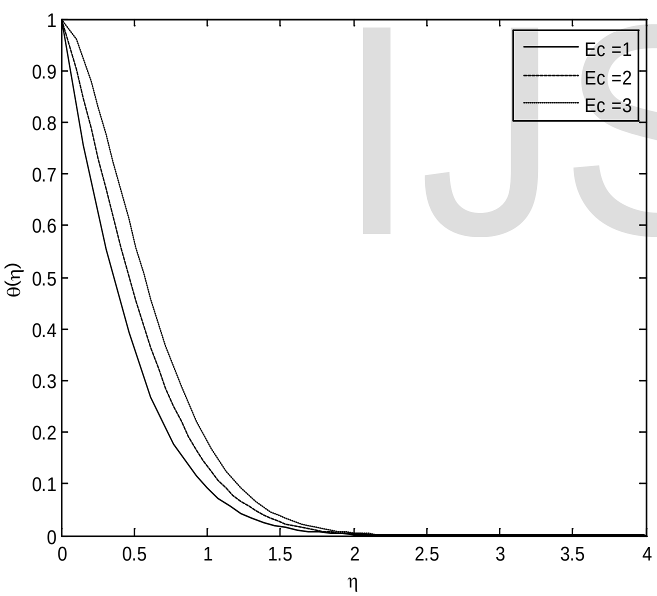

Fig.9 Temperature profiles for different values of Ec

Table 1. Variation in Skin friction coefficient and Nusselt Number

Gr_x	Ec	K	n	S	$f''(0)$	$-\theta'(0)$
1	0.1	0.5	1	1	-1.961008	2.550926
5					-1.19117	2.631503
10					-0.267588	2.704816
	1				-1.689232	1.695381
	2				-1.608964	0.843724
	3				-1.532799	0.108209
		1			-1.595630	2.568079
		2			-1.363862	2.559828
		3			-1.209803	2.553151
			1		-1.765369	2.573178
			2		-2.719053	3.576637
			3		-3.494392	4.429028
				1	-1.765369	2.573178
				2	-2.350731	3.201768
				3	-3.000446	3.908448

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6 CONCLUSIONS

This study reports the numerical solution for thestagnation-point flow of a micropolar fluid towards a nonlinearly stretching surface in presence of viscous dissipation with suction and injection effects. The influence of non-dimensional governing parameters on velocity and temperature profiles along with friction factor and heat transfer rate was discussed and presented through graphs and tables. The conclusions are made as follows:

- Material parameter have tendency to improve the friction factor.
- Increase in suction parameter, buoyancy parameter and the value of n enhances the heat transfer rate.
- Enhancement in viscous dissipation parameter improves the temperature profiles of the flow.
- Increase in suction parameter reduces the friction factor.